GENERATION OF AVAILABLE POTENTIAL ENERGY IN HURRICANE HILDA (1964)

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ABSTRACT

The energetics of hurricane Hilda (1964) are studied through the theory of available potential energy applied to a limited fixed region surrounding the storm. The generation of available potential energy is shown to be closely dependent on differential heating within the baroclinic structure of the hurricane, occurring primarily in the middle and upper troposphere of the core of the storm. The diabatic heating components (condensation, emission of long wave radiation, direct absorption of solar radiation, and sensible heating) are modeled and the contribution to the total generation from each component computed. The results from the latent heating model based on Kuo's work portray the dependence of the deep cumulus convection on the sea surface temperature. The best estimate of the total generation of available potential energy within the hurricane scale is 10.3×10^{12} watts, of which 77 percent is generated by latent heating, 17 percent by infrared cooling, and 6 percent by direct solar absorption. The total generation compares favorably with estimates of kinetic energy production in mature hurricanes. Energy considerations in the steady state condition of the hurricane system are discussed within the framework of the available potential energy theory.

1. INTRODUCTION

The hurricane is comprised of many scales of atmospheric motion. The large-scale circulation is a warm core baroclinic vortex with characteristic wind speeds of 50 m./sec. in the vicinity of the eye wall. This horizontal motion comprises nearly all the kinetic energy in the storm. Superimposed on this horizontal motion is a slow quasi-steady thermally driven direct circulation characterized by inflow in the low level, relatively intense ascending motion near the core, outflow at high levels and slow sinking motion around the fringe of the storm. The radial extent of this circulation is typically on the order of 1000 km.

A second important scale of motion in the hurricane system is the convection near the hurricane's core, as emphasized in the "hot tower" concept by Riehl and Malkus [30]. Although the dimension of this scale is two orders of magnitude smaller than that of the hurricane circulation itself, the vertical motion in the convection provides the vital link between the source of thermal energy at the convective scale and the production of kinetic energy for the large scale.

Recently, Charney and Eliassen [3], Ooyama [24], Kuo [15], and Ogura [23] have attempted to separate the cumulus scale motion from the macroscale motion while retaining the cumulative effect of the latent heat release by the cumulus convection. In this way the large-scale motion is thermally driven by the net heating effect of the many small convective elements.

For the large-scale thermally direct circulation to develop, the cumulus convection must be organized to produce horizontal temperature gradients and baroclinicity associated with the hurricane's warm core. The importance of the warm core in the production of kinetic energy on the large scale was recognized by Palmén [25] and recently reemphasized by Yanai [32]. Yanai shows that baroclinicity is a necessary requirement for the maintenance of kinetic energy in the tropical cyclone since the solenoid field associated with the warm core vortex is in the kinetic energy producing sense, tending to accelerate the direct circulation.

Within the realm of atmospheric energetics, the rising motion in the warm core and subsidence near the fringe of the hurricane imply a conversion from potential to kinetic energy in this region. Both of these conditions suggest studying the hurricane energetics from the framework of available potential energy. In this paper the theory of available potential energy is used to determine the effect of differential heating. Such an approach will delineate the regions of a hurricane where heating will be most important in the production of energy available for conversion to kinetic energy.

2. GENERATION OF AVAILABLE POTENTIAL ENERGY WITHIN THE HURRICANE SCALE

The atmosphere's available potential energy is defined by Lorenz [17] as the difference between the sum of the internal and the gravitational potential energies of a given state of the atmosphere and the sum of the energies which would exist after an adiabatic mass redistribution to a statically stable horizontal density stratification. During this redistribution process the available potential energy is converted to kinetic energy as the warm air rises and the cold air sinks. The source of the available potential energy is the differential heating of the atmosphere by the diabatic processes.

In their recent paper, Dutton and Johnson [5] extended Lorenz' work on the theory of available potential energy by showing that the theory is valid for all scales of atmospheric motion. Dutton and Johnson stress the importance of adding heat at high pressure and extracting heat at low pressure in the generation of available potential energy. This effect, noted by Lorenz [18], is disregarded in studies of the diabatic generation based on the concept that the source of available potential energy is due primarily to isobaric differential heating. Both effects are incorporated in the exact expression for the generation of available potential energy (after Dutton and Johnson [5]) given by

$$G=\int_A\int_{\Theta_0}^{\Theta_T}\left[1-\left(\frac{\overline{\overline{p}}}{p}\right)^\kappa\right]J_\Theta\dot{Q}d\Theta dA \tag{1}$$

where p is the pressure, Θ is potential temperature, J_Θ is the Jacobian of transformation $|\partial h/\partial\theta|$ between height h and potential temperature, $\dot{Q}$ is the rate of heat addition per unit volume and κ is the ratio of the gas constant R to the specific heat at constant pressure c_p. The integration extends horizontally over the entire area A of each isentropic surface and vertically from the lowest potential temperature of the atmosphere Θ_0 to Θ_T equal to infinity at the top of the atmosphere.

Since the mean pressure in (1) is determined by integrating over the entire global isentropic surface, its direct application for the estimation of the generation of available potential energy is only possible when global data are available. Ideally one would prefer to separate the available potential energy associated with each scale of atmospheric motion. However, since this is a formidable problem because of scale interactions, only a scale-wise decomposition of the generation integral is attempted. The local generation associated with a particular scale is based on the concept that the mass distribution may be considered to be a superposition of scales.

For the scale decomposition, the global generation is divided into two portions,

$$G=G_{R_1}+G_{R_2} \tag{2}$$

where

$$G_{R_1}=\int_{A_1}\int_{\Theta_0}^{\Theta_T}\left[1-\left(\frac{\overline{\overline{p}}}{p}\right)^\kappa\right]J_\Theta\dot{Q}d\Theta dA \tag{3}$$

and

$$G_{R_2}=\int_{A_2}\int_{\Theta_0}^{\Theta_T}\left[1-\left(\frac{\overline{\overline{p}}}{p}\right)^\kappa\right]J_\Theta\dot{Q}d\Theta dA. \tag{4}$$

G_{R_1} represents the contribution to the global generation by the atmosphere surrounding the hurricane scale of motion and G_{R_2} is the contribution to the global generation within a region R_2 containing the hurricane.

Over most of the subtropical region the pressure $p(x, y, \Theta)$ equals a mean pressure $\overline{p}(\Theta)$ and only in the tropical depressions, hurricanes, or small-scale convection does the pressure distribution depart significantly from this condition. Thus in the Tropics it seems reasonable to consider that the hurricane is an isolated scale superimposed on the nearly flat horizontal barotropic mass distribution.

From this consideration, G_{R_2} is further divided into two components given by

$$G_{R_2}=\overline{G}_{R_2}+G'_{R_2} \tag{5}$$

where

$$\overline{G}_{R_2}=\int_{A_2}\int_{\Theta_0}^{\Theta_T}\left[\frac{\overline{p}^\kappa-\overline{\overline{p}}^\kappa}{p^\kappa}\right]J_\Theta\dot{Q}d\Theta dA \tag{6}$$

$$G'_{R_2}=\int_{A_2}\int_{\Theta_0}^{\Theta_T}\left[1-\left(\frac{\overline{p}}{p}\right)^\kappa\right]J_\Theta\dot{Q}d\Theta dA. \tag{7}$$

$\overline{G}_{R_2}$ is the generation associated with the barotropic scale and G'_{R_2} is the generation within the hurricane scale due to its baroclinic nature. Mathematically the three generation expressions sum to the original generation integral and the scalewise interpretation of the generation seems justified.

Since we wish to determine if the hurricane is an energetically self-sustaining thermodynamic system, we only investigate in detail G'_{R_2}. If the region R_2 were isolated with no advection through the lateral boundaries, (7) would be the exact expression for the generation of available energy in R_2. However, since R_2 interacts with the region R_1, $\overline{G}_{R_2}$ should be considered. In (6), $(\overline{p}^\kappa-\overline{\overline{p}}^\kappa)$ will be positive below 300 mb. since the average pressure of the Θ surface within the Tropics will be higher than the global average. Thus, a contribution to the generation of available potential energy for the global scale occurs from the net heating within the region.

3. STRUCTURE OF HURRICANE: BAROCLINICITY AND THE EFFICIENCY FACTOR

The effects of the various diabatic processes within the hurricane scale can be studied through the concept of an efficiency factor. Following Dutton and Johnson [5], the efficiency factor for the hurricane scale from (7) is

$$\epsilon=\left[1-\left(\frac{\overline{p}}{p}\right)^\kappa\right]J_\Theta. \tag{8}$$

The physical explanation for the efficiency factor lies in the definition that available potential energy is the dif-

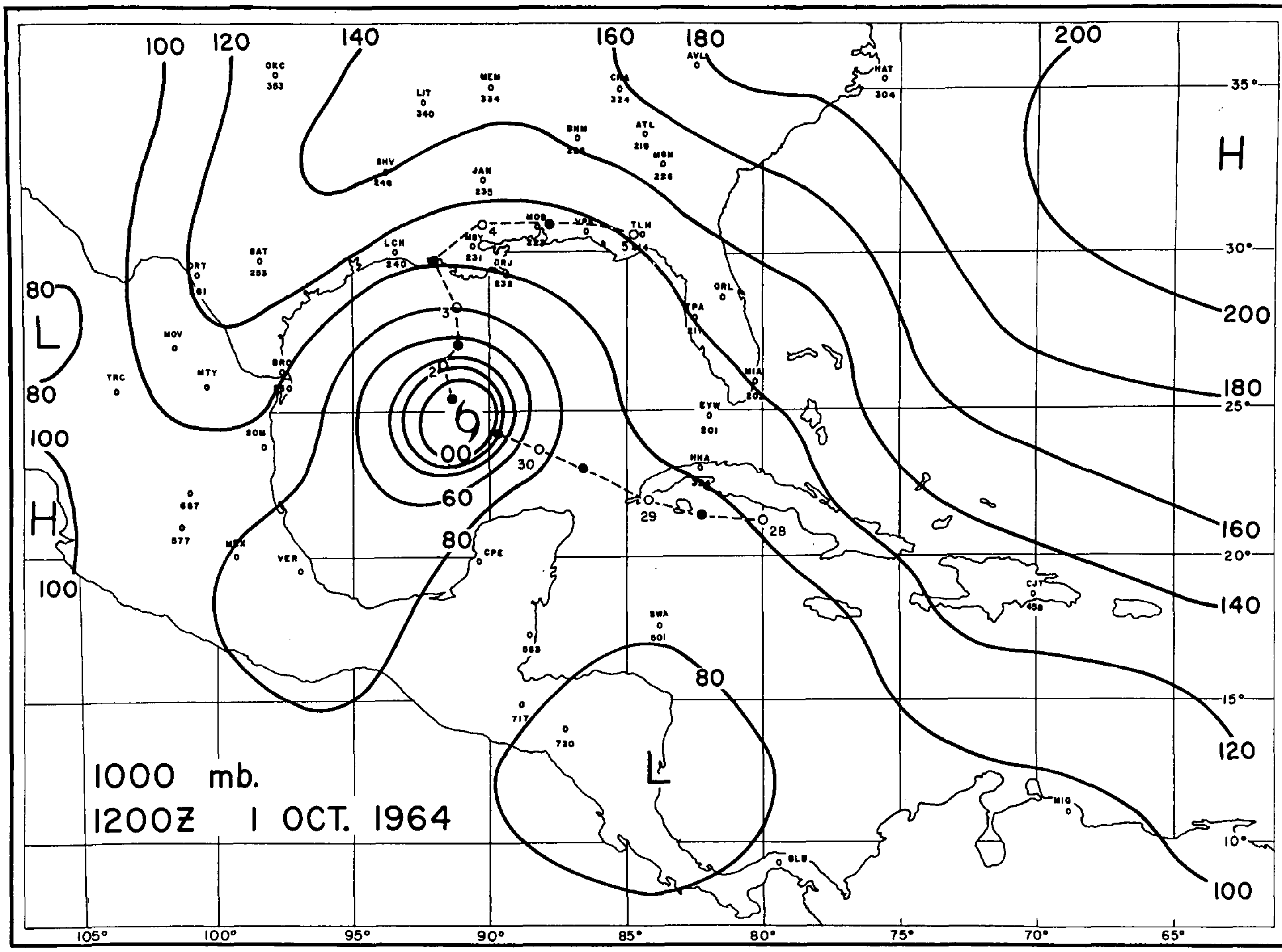

FIGURE 1.—1000-mb. geopotential surface, from Hawkins and Rubsam [10]. Track of Hilda indicated by dashed line. Daily 00 GMT and 12 GMT positions indicated by ● and ○ respectively.

ference of the total potential energy of an actual atmosphere and the total potential energy of the hypothetical reference atmosphere. In the presence of heating or cooling, the total potential energies of both states change by different amounts. The efficiency factor field provides the proper weighting of the heating field to determine the change of this difference, i.e. the change of available potential energy. Note 1) that heating (cooling) at high pressure and cooling (heating) at low pressure relative to the mean pressure of an isentropic surface within the region creates (destroys) available potential energy and 2) that the distribution of ϵ illustrates quantitatively where heating and cooling will be effective in the generation of available energy. To apply this concept, an efficiency factor cross section for hurricane Hilda (1964) was constructed from the available data.

On Oct. 1, 1964, at 12 GMT hurricane Hilda was located at 25°N., 91°W., near the center of the Gulf of Mexico as shown in figure 1 and the TIROS 7 satellite photograph (fig. 2). An area extending radially 1000 km. from the center of the storm in all directions was selected to be the representative region R_2 for the storm. Data for this region were provided mainly from three flights of ESSA Research Flight Facility planes at approximately 800, 600, and 200 mb. Conventional radiosonde data around the Gulf of Mexico supplemented the aircraft data.

From these data end mean soundings, for the area [12], an east-west potential temperature and efficiency factor cross section through the region R_2 enclosing Hilda was constructed (fig. 3). In the warm core, pressures are higher than the isentropic average of the hurricane region and the positive efficiency factors are large. In the outer region of the storm, pressures are lower and the efficiency factors are negative.

As expected, the magnitude of the efficiency factors is also dependent on the size of the region over which the mean pressure of each Θ surface is determined. How-

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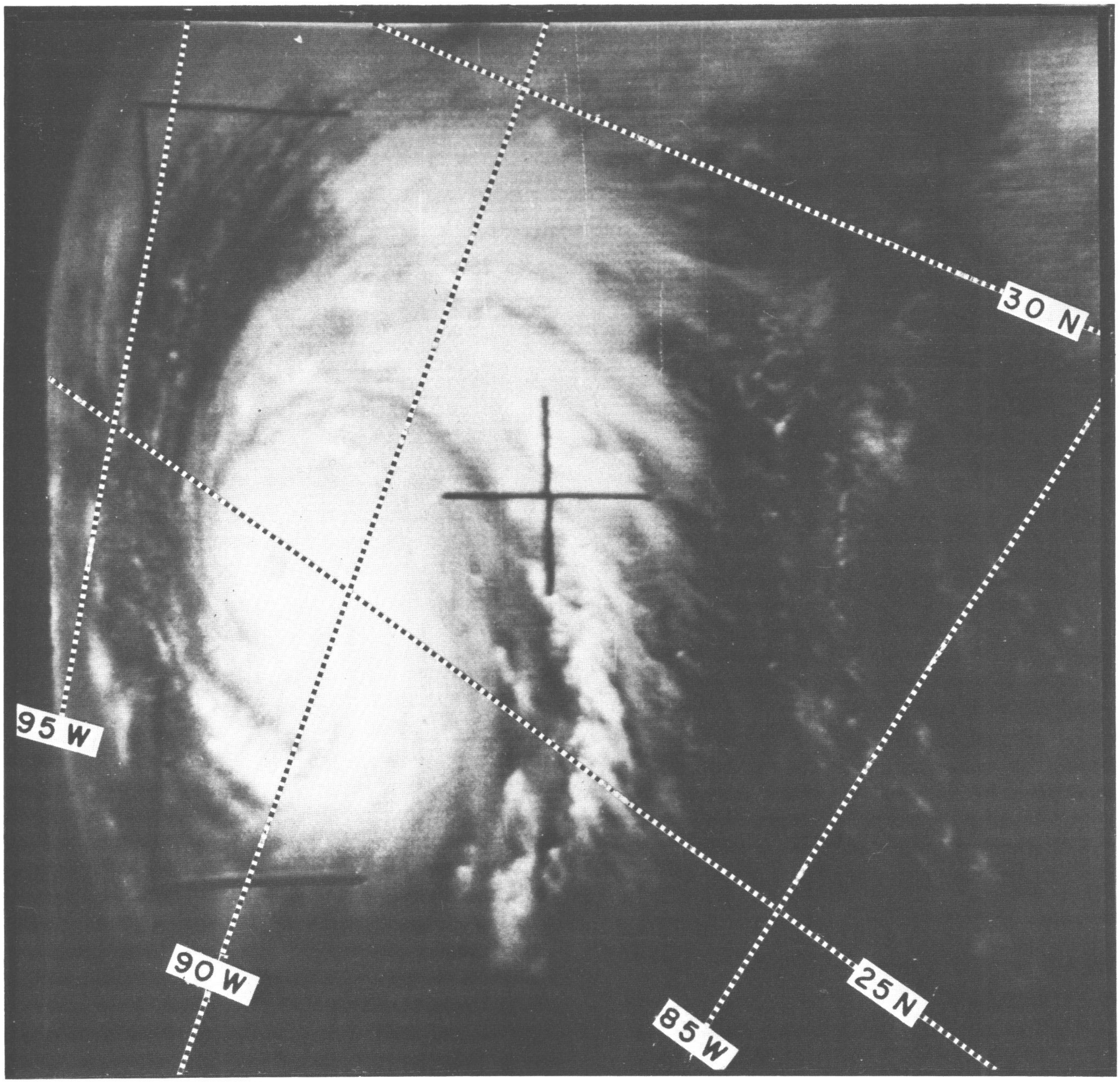

FIGURE 2.—TIROS 7 photograph of hurricane Hilda, 1456 GMT Oct. 10, 1964, pass 6952/51.

ever, the dependence is slight. Beyond a certain radial distance the change of $\overline{p}(\theta)$ by enlarging the horizontal extent of the region R_2 is extremely small since $\overline{p}(\theta)$ is nearly equal to the pressure $p(x, y, \theta)$ in the added region. To study the influence of the areal extent, efficiency factors were computed again over a region one-fourth the size, i.e., an area with a radius of 500 km. The reduction in size affected the maximum efficiency factors by no more than 20 percent and shifted the zero line inwards about 200 km. Since we are mainly interested in the differential effects of heating, these differences do not appear to be significant.

Another important element in the relationship among the efficiency factor, baroclinicity, and heating is the tendency for a "feedback" mechanism. On an isentropic surface heating at high pressure, the positive efficiency area, results in a steeper slope of the isentropic surfaces in the region separating the cold and warm air, an increase in the baroclinicity, and a creation of additional available potential energy. Conversely, cooling at high pressure results in a decreased slope of the isentropic surfaces, a decrease in the baroclinicity, and a destruction of available potential energy. Thus heating distributions that tend to increase the baroclinicity also increase the magnitude of

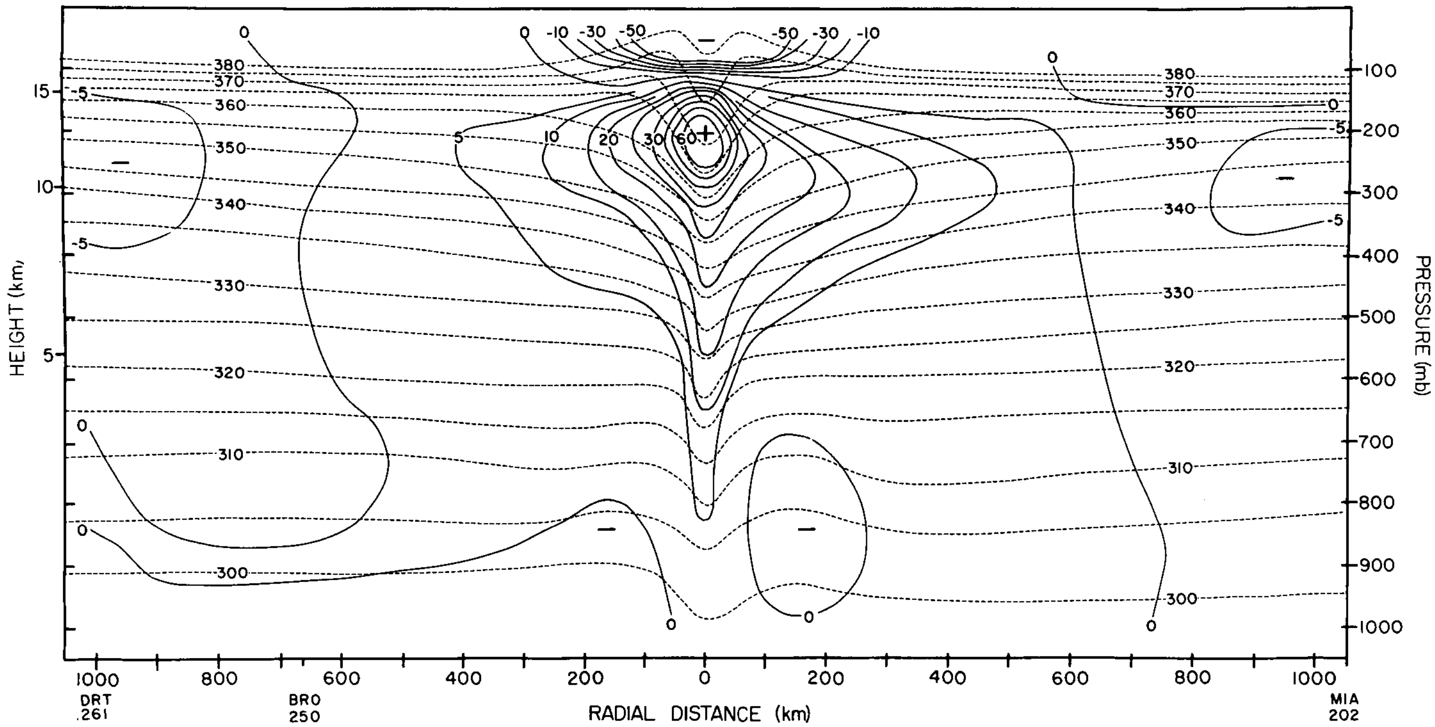

FIGURE 3.—Potential temperature (°K.) and efficiency factor (m./deg.) cross section through Hilda, Oct. 1, 1964.

the efficiency factors and subsequent heating and cooling are even more efficient in producing available potential energy. The converse is true for a decrease in the baroclinicity.

4. AVAILABLE POTENTIAL ENERGY GENERATION BY THE COMPONENTS OF HEAT ADDITION

In this section estimates of the generation of available potential energy within Hilda by the diabatic processes are made. The five physical processes of heating in the atmosphere are: 1) release of latent heat, 2) emission of long wave radiation, 3) direct absorption of solar radiation, 4) sensible heat transfer through the earth-atmosphere interface, and 5) frictional dissipation of mechanical energy. In the boundary layer where frictional dissipation is high, efficiency factors are quite small, indicating that the direct generation contribution from this source will be slight. In the free atmosphere, the generation contribution from internal friction is possibly weakly positive since higher values may be expected in the inner region of positive efficiency factors where the kinetic energy and the vertical shears are greatest. However, because of the positive and negative efficiency factor weighting, the generation within the entire hurricane region by frictional dissipation may be presumably neglected. From these considerations only the first four processes are modeled and their contributions to the total generation of available energy are computed.

RELEASE OF LATENT HEAT

As is generally emphasized, the primary source of energy in the hurricane is the release of latent heat through cumulus convection. It is evident that both the vertical and the horizontal distribution of the components of heat addition as well as the efficiency factor distribution must be known to determine the generation of available potential energy. Two methods are developed to estimate the spatial distribution of latent heating. The first is a simple statistical cloud model and the second is a more sophisticated one based on Kuo's [15] model for latent heat release.

FIRST METHOD FOR LATENT HEAT RELEASE

In the first model surface air is lifted dry adiabatically to the lifting condensation level, then essentially moist adiabatically to 100 mb. During the moist adiabatic process the cloud parcel properties are modified every 100 mb. by entraining environmental air. Since this results in a temporary desaturation, the resultant parcel lapse rate lies between the dry and moist adiabatic lapse rate. Under the assumption that condensation occurs when the relative humidity reaches 100 percent and that all condensed water vapor falls out of the system, the amount of latent heat released within each 100-mb. layer is approximated by

$$\Delta H = L\Delta q\bar{\rho} \tag{9}$$

where ρ is the mean density of the layer. Δq, the difference of the specific humidity between the top (t) and bottom (b) of each layer, is defined by

$$\Delta q = q_{s_t} - [(1-\alpha) q_{s_b} + \alpha q_b] \tag{10}$$

where α is the entrainment percentage (20 percent in this initial study [2]), q_s is the saturation specific humidity and q is the environmental specific humidity.

The heating rate $\dot{Q}$ in (7) is obtained from the release of latent heat within the convective clouds ΔH, the time dependency of the heat release, and the percentage of area covered by convective clouds. If the percentage area of cloud cover in an incremental radial ring is B and the time for the cloud to ascend 100 mb. is Δt, the average rate of heat addition becomes

$$\dot{Q} = B\Delta H / \Delta t. \tag{11}$$

Assuming the parcel's ascent rate to be linear with respect to pressure, Δt is determined by the lifetime of the cloud, i.e., the time required for the cloud to ascend from the point of initial condensation to the 100-mb. level. Based on Ogura's [23] estimates, the cloud lifetime was set equal to 60 min. This yields a Δt of 6.6 min., which is higher than Gray's [9] estimate of 30 min.

The following percentages of areal cloud coverage by cumulonimbi based on Gentry's [8] study of rainbands within a hurricane were assigned:

Radius (km.)	*B—percent areal coverage*
0–100	15
100–200	10
200–300	4
300–400	3
400–500	1

SECOND METHOD FOR LATENT HEAT RELEASE

The second method of estimating the latent heat release is adapted from Kuo [15]. This approach assumes that the latent heat release is determined by the flux of moisture from the boundary layer induced dynamically by the hurricane. The source of moisture is evaporation into the boundary layer and frictional cross-isobaric inflow of moist air.

In parameterizing the rate of latent heat release, Kuo introduces the concept of "rate of cloud production," defined as the ratio of the net moisture convergence in a vertical column, I, to the amount of moisture, Q, that is needed to saturate and warm the column to a saturation specific humidity q_s and temperature T_s. If the differential horizontal advection of moisture is negligible in comparison with the vertical flux of moisture from the boundary layer, I is given by

$$I = \begin{cases} -\omega_0 q_0 g^{-1}, & \omega_0 < 0 \\ 0, & \omega_0 \geq 0 \end{cases} \tag{12}$$

where ω_0 and q_0 are the individual derivative of pressure and specific humidity at the top of the boundary layer, and g is the acceleration of gravity. Q is given by

$$Q = g^{-1} \int_{p_0}^{p_t} (q_s - q) dp + g^{-1} \int_{p_0}^{p_t} c_p L^{-1} (T_s - T) dp \tag{13}$$

where p_0 is the pressure at cloud base and p_t is the pressure at cloud top.

In Kuo's model the rate of heat addition to the environment is

$$\dot{Q} = \rho c_p (T_s - T) \frac{I}{Q}. \tag{14}$$

T_s and q_s are given by the moist adiabat through the lifting condensation level; T and q are known from the aircraft data and ω_0 is modeled after Ogura [23].

In Ogura's model the radial profile of ω_0 is estimated from the surface stress and the absolute vorticity and given by

$$\omega_0 = -\frac{g}{r} \frac{\partial}{\partial r} \left[r\tau_0 \Big/ \left(f + \frac{\partial (rV_0)}{\partial r} \right) \right] \tag{15}$$

where τ_0 is the surface stress, V_0 is the tangential wind speed at the top of the boundary layer, and f is the Coriolis parameter. An empirical model for the radial distribution of the tangential wind speed of the form, $V_0 r^{1/2} = C$ was assumed, with the constant C determined by a least squares fit of the model to the observed winds beyond the radius of the maximum tangential wind (Anthes [1]). The solution for the steady state mean vertical motion from equation (15) (fig. 4) and this model was used, and realistic estimates of the latent heating rate were determined.

COMPARISON OF LATENT HEAT RELEASE AND GENERATION RESULTS

The sea surface temperature is a parameter that has been suggested as being important in hurricane formation. Palmén [25] noted that the temperature must exceed a critical value of 26–27°C. The effect of this criterion on the intensity of the hurricane's circulation can be

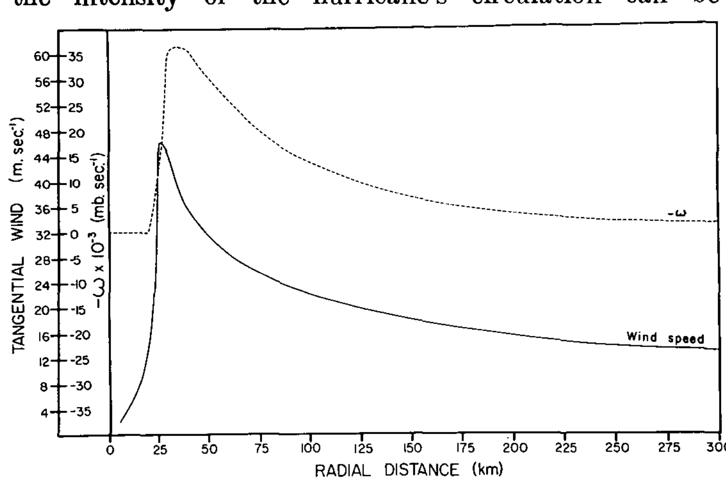

FIGURE 4.—Tangential wind speed and vertical motion computed from analytical model.

examined in both models by studying the generation due to changes in the latent heat release associated with different initial values of potential temperature of the cloud parcel that originates from the boundary layer. The other important boundary layer parameter, the specific humidity, is related to the sea surface temperature by noting that the boundary layer relative humidity tends to remain between 80 and 90 percent (Dunn and Miller [4]). A fixed relative humidity of 85 percent is used to provide unique temperature-specific humidity dependency so that the effect of varying only the surface temperature can be studied. All other parameters are held constant.

In this paper, 27°C. is selected as the reference (Case II) to study the critical ocean temperature, and changes in heat release that result under conditions 3° cooler (Case I) and warmer (Case III) than this reference are investigated.

The radial and vertical distribution of the latent heat for Cases II and III using both methods are shown in tables 1 and 2. A corresponding table for Case I is presented by Anthes [1].

Note for the second model the large region in which there is little or no latent heat release, while in the first model relatively large amounts of latent heat are always released throughout the system. In Case III the striking difference between the two models is lacking. Similarly the results for the total latent heat release due to condensation within the 1000-km. region presented in table 3 summarize this important difference. The difference in the results of the two methods for Cases II and III indicates that the amount of latent heat released is sensitive to the surface temperature in the second method but relatively insensitive in the first method. The greater sensitivity in the second method is due to the emphasis on the temperature of the environment with respect to the temperature of the ascending cloud in determining the maximum height of convection and the amount of moisture available for release as latent heat. It is interesting to note that for a fixed sea surface temperature, according to this model, larger (smaller) amounts of latent heat would be released if the environment were cooler and more moist (warmer and drier). Thus, model two stresses that the surface temperature is not the sole temperature parameter in determining the heat released, but the critical condition involves the difference of the equivalent potential temperature of the surface air and the potential temperature distribution of the environment.

The first model fails to be sensitive to the surface temperature because there is no control by the environment on the convection since the area cloud coverage B and cloud lifetime Δt are fixed. In this model the rate of convection is implicitly fixed and the cloud parcel always penetrates to 100 mb. The latent heat released in the three cases is nearly identical. The quantitative effect of varying the parameters B and Δt is evident since they are linearly related to the heating and generation. For example, if Gray's [9] estimate of 30 min. for the cloud lifetime is used the generation is doubled. However, if this estimate is combined with Malkus' et al. [21] estimate for cloud coverage, which is approximately one-half of Gentry's, the generation estimates are identical to the results presented.

A deficiency in the second model is the use of a technique designed for obtaining the cumulative effects of latent

TABLE 1.—*Latent heat release and generation for moderate case (T_0=27° C.)*

Pressure (mb.) \ Radial distance (km.)	500–400	400–300	300–200	200–100	100–0
100–200	.02 .02 0 0	.06 .06 0 0	.10 .11 0 0	.34 .26 0 0	1.2 1.8 0 0
200–300	.19 .08 0 0	.60 .36 0 0	.92 .71 .10 .10	2.9 4.0 2.3 3.0	7.4 25. 2.1 7.
300–400	.54 .20 .30 .08	1.7 .77 .50 .20	2.5 1.9 .90 .60	7.4 9.3 4.9 6.0	14. 40. 4.4 10.
400–500	.88 .09 .5 .06	2.7 .55 1.0 .20	3.8 1.4 2.0 .8	11. 6.8 6.2 4.0	18. 26. 12. 19.
500–600	1.1 .11 .5 .04	3.4 .55 .8 .1	4.7 1.0 2.3 .4	13. 3.1 10. 2.0	20. 18. 23. 21.
600–700	1.3 .09 .3 .02	3.8 .34 .7 .07	5.4 .52 2.3 .2	15. 1.4 13. 1.0	22. 11. 46. 25.
700–800	1.5 .15 .3 .03	4.5 .38 .5 .04	6.1 .39 2.6 .20	17. .54 16. .50	24. 7.9 35. 9.7
800–1000	2.6 .02 .6 0	8.0 .06 1.3 .01	12. 0 5.0 0	33. −.2 18. −.1	49. 6.5 71. 6.2

Plotting model	
$\dot{Q}_1$	G_1
$\dot{Q}_2$	G_2

$\dot{Q}_1$: latent heat release method 1 (10^{-2} watt m.$^{-3}$)
G_1 : generation from method 1 (watt m.$^{-2}$)
$\dot{Q}_2$: latent heat release from method 2 (10^{-2} watt m.$^{-3}$)
G_2 : generation from method 2 (watt m.$^{-2}$)

TABLE 2.—*Latent heat release and generation for warm case (T_0=30° C.). Plotting model legend same as table 1*

Pressure (mb.) \ Radial distance (km.)	500–400	400–300	300–200	200–100	100–0
100–200	.02 .02 0 0	.08 .08 0 0	.13 .14 .04 .04	.47 .35 1.4 1.0	1.5 2.3 7.8 17.
200–300	.24 .10 .32 .10	.78 .47 .84 .50	1.2 .93 2.3 1.9	3.7 5.1 7.7 12.	8.9 30. 22. 76.
300–400	.66 .21 .60 .20	3.0 .95 1.4 .60	3.1 2.4 3.2 2.4	8.6 11. 9.5 11.	16. 45. 23. 55.
400–500	1.0 .10 .77 .1	3.2 .64 1.6 .40	4.4 1.6 3.8 1.5	12. 7.6 10. 6.3	19. 28. 28. 46.
500–600	1.3 .12 .73 .06	3.9 .63 1.5 .2	5.3 1.2 4.0 .7	14. 3.4 13. 2.9	22. 19. 34. 31.
600–700	1.4 .10 .62 .04	4.3 .45 1.5 .1	6.0 .59 4.0 .4	16. 1.6 14. 1.3	23. 12. 41. 23.
700–800	1.6 .06 .63 .03	5.0 .43 1.4 .10	6.8 .44 4.3 .3	19. .6 16. .6	26. 8.4 43. 12.
800–1000	2.8 .01 .08 0	9.0 .03 1.9 .01	13. 0 5.2 0	34. −.2 18. −.1	51. 4.2 51. 1.9

TABLE 3.—*Total latent heat release (10^{14} watts) and generation due to latent heat release (10^{12} watts)*

T_0		First method		Second method	
		Heating	Generation	Heating	Generation
Case I	24°C.	4.1	7.3	0.7	0.4
Case II	27°C.	4.6	8.5	2.5	5.3
Case III	30°C.	5.2	10.4	4.4	13.0

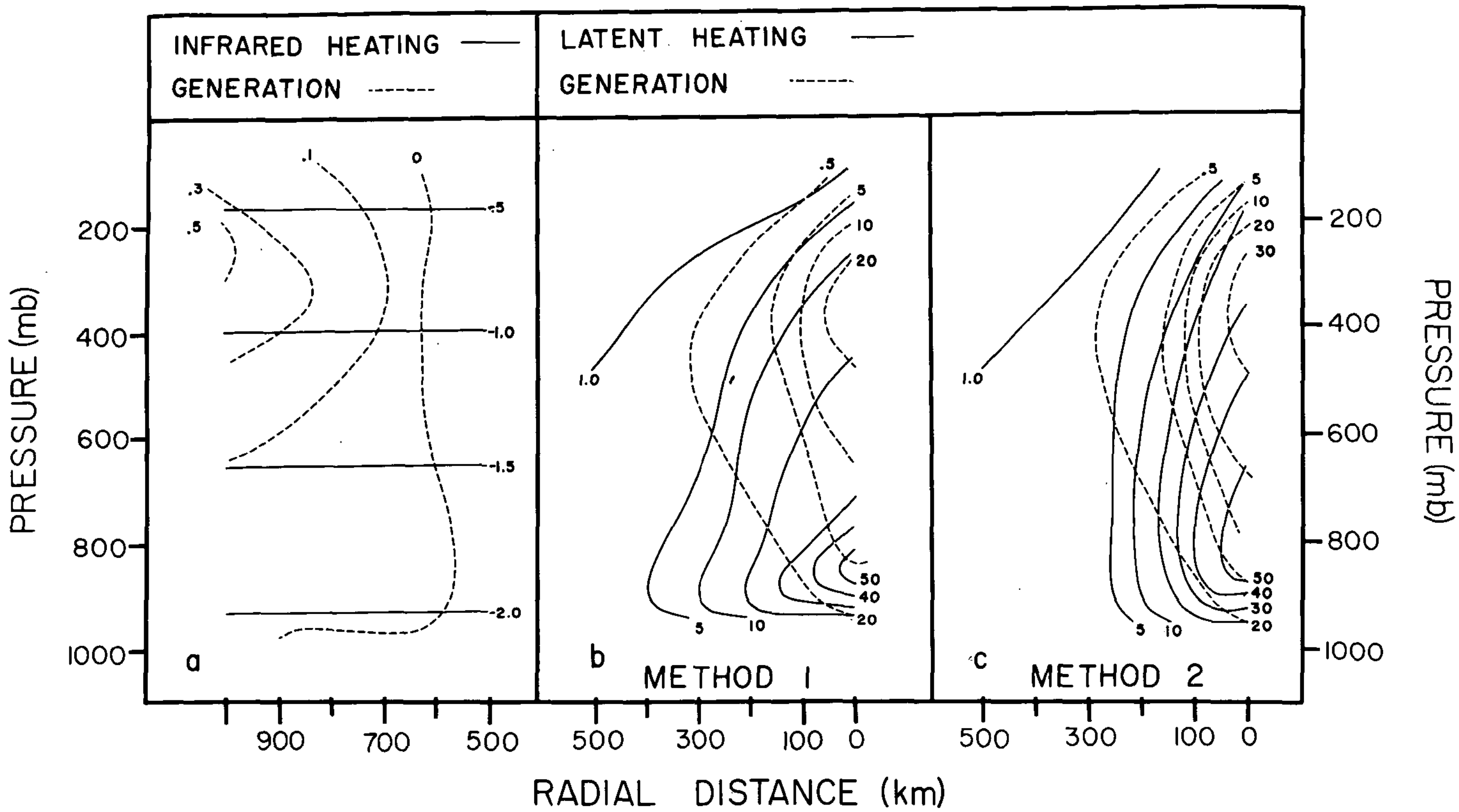

FIGURE 5.—Heating (10^{-2} watts m.$^{-3}$) and generation (10^{-3} watts m.$^{-3}$) rates due to infrared cooling and latent heating (Case III).

heat through the process of free convection while in certain stages of a hurricane the release of latent heat through forced convection may be significant. The use of Kuo's model in a case of forced convection in an unsaturated environment in which $T_s \approx T$ is invalid. In this case the numerator of (14) is zero while the denominator is finite, and the heating rate is underestimated. In the 24°C. sea surface case the difference between T_s and T is small; hence the estimates of latent heat release and generation are possibly underestimated if forced convection is significant.

In spite of the possible deficiencies, the total latent heat release for the higher temperatures for both models compares favorably with Riehl's [28] estimate of $2\text{–}5\times10^{14}$ watts for a mature storm, and Hughes' [11] result of 5×10^{14} watts for 28 Pacific tropical cyclones. In future experiments it would be interesting to extend the dependency of the generation to other parameters in addition to the sea surface temperature by using models which are more realistically based on cloud dynamics, such as the model proposed by Malkus [19].

BEST ESTIMATE OF GENERATION DUE TO LATENT HEAT

Measurements of the water temperature immediately before Hilda's passage through the Gulf of Mexico indicated fairly uniform temperatures at about 29–30°C. (Leipper [16]). However, Leipper indicates that the sea surface temperature lowered to 28°C. during the hurricane's passage due primarily to mixing of the Gulf's surface layer by the wind. Thus, an extrapolated value of 3.2×10^{14} watts for latent heat release from the results for Cases II and III of the second method is probably the most realistic estimate of the latent heat release.

The distribution of latent heat from the three cases for both models was used, and the generation of available potential energy within the region R_2 was estimated using a finite evaluation of (7). The heating rates and generation results are presented for Cases II and III in tables 1 and 2 and summarized for all three cases in table 3. The vertical and horizontal distribution of latent heating and the associated generation are presented in figure 5.

In the results, it is clear that the greatest generation occurs within 100 km. of the eye in the middle troposphere. In this region the efficiency factors are high and the latent heat release is large. As the radial distance increases, the efficiency factors and latent heat release decrease rapidly, so that over 50 percent of the generation occurs in the inner 100 km.

The vertical distribution of generation shows that approximately 90 percent occurs in the middle troposphere between 700 and 200 mb. and is associated with large positive efficiency factors. Below 700 mb., even though the latent heat release is strong, the efficiency factors are smaller and there is little direct generation. Above 200 mb., the release of latent heat is relatively unimportant.

In a comparison of the results for the 30°C. case, the generation estimates are greater for the second model even though the latent heat release by the first model is larger. This difference emphasizes that not only the amount but also the distribution of the heat release is important in the generation of available potential energy.

Therefore, we conclude 1) that although both methods give reasonable results for the warm case (III), the second method is physically more realistic and 2) that the best estimate of the generation due to latent heating is an interpolated value of 7.9×10^{12} watts associated with a sea surface temperature of 28°C.

EMISSION OF LONG WAVE RADIATION

In the hurricane's cross section, the magnitude of the negative efficiency factors is small; however, the large amount of mass with negative values suggests that generation by the infrared component should be investigated.

The infrared radiation model for this study was formulated from actual radiometersonde data and a typical cloud distribution for a hurricane. Several radiometersonde flights with typical subtropical cloud distributions were selected from a series of approximately 100 flights conducted in the Carribean during the last decade. The radiometersonde and the filtering technique used in processing the data are described by Suomi and Kuhn [31] and Kuhn and Johnson [14].

Use of a typical cloud distribution in a hurricane in estimating the generation by the infrared component is justified by the fact that the modification of the atmosphere's infrared irradiance is determined primarily by the cloud distribution and secondarily by the moisture distribution. In a typical hurricane extensive multileveled cloudiness with cumulonimbi penetrating to the tropopause occurs in the central region of the storm. A cirrus deck near the tropopause covers this region. From both radiometersonde evidence and theoretical considerations, such a region under the cirrus shield is nearly in radiational equilibrium and the infrared cooling rate is near zero. However, above the cirrus shield strong cooling of 4–6°C. per day occurs. At some distance from the center, the hurricane is characterized by a sharp break in cloudiness beyond which practically clear conditions prevail (Fett [6]). In this region the infrared cooling may be typified by the "clear air" rate of 1–2°C. per day.

The relationship between the heat addition rate per unit volume and the instantaneous change of temperature $\partial T/\partial t_I$ is

$$\dot{Q}=c_p\rho\frac{\partial T}{\partial t_I}. \tag{16}$$

The infrared component of the generation was computed by using (16) and (7). However, because the exact infrared cooling was not known, a satellite picture was used to indicate cloud coverage and the generation was computed for the extremes of a range of cooling rates.

The satellite picture (fig. 2) of hurricane Hilda indicates that beyond 500 km. the region R_2 is free of significant layered clouds; hence the "clear air" cooling rate of 1 to 2°C./day was used for the representative range in this region. For the inner 500-km. region above the cirrus shield a range of 4 to 6°C./day was used while under the cirrus shield the range was 0.0 to 0.3°C./day. Furthermore, since the vertical gradient of efficiency factors in the upper troposphere was large, the infrared generation was computed under two different cirrus distributions, one with the top of the cirrus at 100 mb. where the mean efficiency factors are weakly positive, and the other with the top at 200 mb. where the efficiencies are strongly positive.

From various combinations of cooling rates the total infrared generation ranges between 0.6×10^{12} watts (combination yielding the minimum) and 2.7×10^{12} watts (combination yielding the maximum). Taking the mean cooling rate for each region and assuming the more likely cirrus level of 100 mb., the best estimate for the generation due to the infrared heating component is 1.8×10^{12} watts. The significant contribution to the infrared generation is from the outer "clear" regions of the hurricane environment in the upper troposphere, as shown in figure 5. This is due to the dominating effect of the large amount of mass and the small negative efficiency factors over the large positive efficiency factors and small amount of mass in the inner area.

In a comparison of the infrared generation with the latent heat component associated with the sea surface temperature of 28°C., the ratio of the infrared to the latent heat generation is 1:4. These results indicate that the infrared generation represents a significant source of available potential energy and the cooling in the descending branch of the hurricane circulation is an important feature.

DIRECT ABSORPTION OF SOLAR RADIATION

Since the atmosphere is nearly transparent to direct solar radiation, the generation by this component is presumably small. A simple calculation is made to establish its order of magnitude. From Korb and Möller [13] representative extremes for the range of direct solar heating in cloudy moist regions would be 0.5 to 1.0°C./day while in a clear dry atmosphere values range from 0.1 to 0.3°C./day. Again combining extremes from the two regions of differing cloud conditions the maximum solar generation is 0.96 and the minimum is 0.13×10^{12} watts while the best estimate associated with the mean heating rate in each region is 0.55×10^{12} watts. The ratio of the solar to the latent heating component is 1:15 while compared to the infrared component the ratio is 1:3.3.

SENSIBLE HEAT ADDITION

The fourth generation component studied is that due to sensible heat addition at the earth's interface. The occurrence of both positive and negative efficiency factors in the boundary layer implies that there is both local destruction and production of available energy by the component of sensible heat addition.

Malkus and Riehl [20] show that the process of interface sensible heat transfer does not become effective until the air-sea temperature difference is 2°C. or greater. In outer portions of the region, R_2, it is reasonable to assume that there is little sensible heat transfer since there will be essentially no difference in temperature across the interface. However, in the inner region the air flows horizontally toward lower pressure and tends to cool by adiabatic expansion. Observations show that in spite of the tendency for adiabatic cooling, the temperature remains constant throughout the inflow layer, thus indicating that sensible heat is added. Since observational evidence further indicates that the major reduction in surface pressure occurs in the inner 150 km. of the storm, significant adiabatic expansion and sensible heat addition should occur mainly in this region.

Hawkins and Rubsam [10] calculated a total of 0.15×10^{14} watts for the sensible heat addition in the ring bounded by the 20- and 150-km. radii from the center of Hilda. If this sensible heat is distributed uniformly throughout a boundary layer 1.1 km. in depth, the heating rate per unit volume is 0.204 watts m.$^{-3}$ These results are in excellent agreement with the Malkus-Riehl [20] estimate of 720 Ly/day sensible heat addition over a depth of 1.1 km. in the inner 90 km. of a moderate hurricane, which yields a total of 0.11×10^{14} watts.

When the Hawkins-Rubsam estimate of 0.204 watts m.$^{-3}$, a mean efficiency factor of 1.5 m./deg., and a $\Delta\Theta$ of 3°C. for a boundary layer 1.1 km. in depth over a region 150-km. radial extent are used, a crude estimate for the sensible heat generation is 6.5×10^{10} watts. Because of the choice of a large efficiency factor and this estimate is probably high. Even so, it is two orders of magnitude smaller than the generation components of latent heat and radiation indicating that sensible heat addition is not important in the direct generation of available energy.

The role of sensible heat addition within the framework of the theory of available potential energy is indirect. As the air flows toward lower pressure, sensible heating serves to raise the equivalent potential temperature and is a necessary element in the maintenance of convection surrounding the eye. Ultimately, the latent heat release occurring in the air with the higher equivalent potential temperature maintains the warm core, baroclinicity, and the high efficiency factors, thus making the release of latent heat in this region more effective in the direct generation of available potential energy. Although the sensible heat generation is small, the role of sensible heating in maintaining the core of high equivalent potential temperature is an essential process in the energetics of hurricanes.

TOTAL HEAT BUDGET AND GLOBAL GENERATION CONTRIBUTION

After the latent heat estimate of 3.1×10^{14} watts, the infrared heating estimate of -3.7×10^{14} watts from a mean cooling rate of 1.2°C./day, the solar heating estimates of 0.9×10^{14} watts from a mean heating rate of 0.3°C./day, and the sensible heat estimate of 0.15×10^{14} watts, the net heat addition is 0.45×10^{14} watts. This corresponds to a mean heating rate of about 0.2°C./day unless sensible heat is transported from the region of the hurricane. Riehl's [29] results, although for a smaller volume in the Gulf, indicate that significant sensible heat transport away from a tropical storm does occur. However, in this study the near equality of the two dominating components, infrared cooling and latent heating, indicates the tendency for the hurricane to operate thermodynamically as a closed system over the region R_2.

As mentioned earlier, the net heating within R_2 would provide a positive contribution to the generation of available potential energy for the global scale in addition to the generation for the hurricane scale. After estimates of $\bar{\bar{p}}(\Theta)$ from Dutton and Johnson [5] and the $\bar{p}(\Theta)$ from this study, tentative estimates were made from (6) for $\overline{G}_{R_2}$. The generation contributions from the four heating components for the 28°C. case, method two, are latent heat 9.8×10^{12} watts, infrared -8.0×10^{12} watts, solar absorption 1.6×10^{12} watts, and sensible heating 0.9×10^{12} watts. Thus the total generation contribution for the global scale of 4.3×10^{12} watts or an average of 1.4 watts m.$^{-2}$ from $\overline{G}_{R_2}$ is significant.

5. TOTAL GENERATION WITHIN HURRICANE HILDA AND KINETIC ENERGY PRODUCTION

The total generation for the hurricane scale, G'_{R_2} given by the sum of the components of latent heating, infrared cooling, direct solar absorption, and sensible heating is given in table 4 for both models of latent heat release and the three surface conditions. Our interpolated estimate for total generation rate associated with the Gulf sea surface temperature of 28°C. is 10.3×10^{12} watts which may be compared with estimates of kinetic energy produced in tropical storms, summarized in table 5.

Because of the different areas involved, direct comparisons are difficult. However, it is clear that the total generation of 10.3×10^{12} watts compares favorably with other investigators' estimates. The close agreement between the generation estimate and the estimate for kinetic energy production and advection for Hilda (Hawkins and Rubsam [10]) is especially interesting considering the independent nature of the two methods.

While the numerical comparisons are striking, the physical validity of using computed values of the generation of available potential energy associated with the baroclinic component to estimate also the kinetic energy production depends on the condition that this component of global generation is actually converted into kinetic energy within the system itself. Several observational results in addition to the numerical results indicate that this is true and give validity to the argument that the baroclinic mass distribution of the hurricane may be treated as a scale superposition on the flat barotropic distribution of the Tropics. Among these are: 1) the rela-

TABLE 4.—*Total generation of available energy (10^{12} watts)*

Case	T_0	Total using first model of latent heat	Total using second model of latent heat
I	24°C.	9.7	2.8
II	27°C.	10.9	7.7
III	30°C.	12.8	15.4

TABLE 5.—*Kinetic energy production in hurricanes*

Author	Region of storm (km.)	Kinetic energy production (10^{12} watts)
Gentry [7]	0–110	3.2
Miller [22]	0–110	8.0
Riehl and Malkus [30]	0–110	5.2
Palmén and Jordan [26]	0–220	4.6
Palmén and Riehl [27]	0–666 (entire storm)	15.0
Hawkins and Rubsam [10] for hurricane Hilda	0–150	8.6

tively quiescent state of the hurricane's barotropic environment, 2) the direct circulation associated with the ring of subsidence surrounding the cirrus shield and the ascending branch near the eye, and 3) the slow erratic movement of most hurricanes indicating negligible momentum exchange with the environmental flow. Of the three supporting facts the ring of clear subsiding air indicates the existence of the direct circulation and provides the best evidence that the thermodynamic concept of the generation of the available potential energy associated with the hurricane's baroclinic component is valid. The sinking of the cold environment and the rising of the warm air is the mechanism by which the locally generated available potential energy is continually transformed into kinetic energy. The diabatic components of latent heat and infrared cooling maintain the baroclinicity of the hurricane and the store of available potential energy against the adiabatic conversion process. Furthermore, if the hurricane is a self-sustaining scale of motion, the direct generation within the system must be the primary source of the kinetic energy to offset the sink of frictional dissipation.

The total generation for the three cases for the second method provides an interesting result which indicates the importance of high equivalent potential temperatures in the boundary layer. In Case I with a 24°C. surface temperature, the generation of 2.8×10^{12} watts is insufficient to maintain a well developed hurricane, in Case II with a 27°C. surface temperature the generation of 7.7×10^{12} watts is probably sufficient but marginal, while in Case III with a 30°C. surface temperature the generation is sufficient to maintain an intense hurricane. The actual dynamics of the generation are related to the difference of the environment's potential temperature and the equivalent potential temperature of the surface air. Quite likely for a characteristic potential temperature for the environment, there is a corresponding sea surface temperature for which the hurricane tends to exist in a steady state, in which the direct generation, the rate of kinetic energy production and the frictional dissipation are nearly equal. An imbalance between the three processes will result in a change of intensity in the storm. For example, if the hurricane moves over a warmer sea surface, the model indicates that the generation will increase. Unless the adiabatic cooling due to the direct circulation immediately increases, thus compensating for the increased rate of latent heat release, greater baroclinicity and higher efficiency factors are the immediate result. The increased baroclinicity is important in that the associated increased thermal wind component eventually allows for higher wind speeds and greater amounts of kinetic energy to be maintained in a balanced state in the hurricane system. The increased kinetic energy results from an increase in the rate of production of kinetic energy over that dissipated by friction. Eventually the dissipation of kinetic energy will also increase due to the greater shears in both the boundary layer and free atmosphere until a new steady state condition is reached in which the generation, production of kinetic energy, and frictional dissipation are again nearly equal, but are all greater than the rates corresponding to the lower sea surface temperature.

7. SUMMARY

In this study the estimates of the generation of available potential energy associated with the hurricane's baroclinicity from four components of heat addition have been determined. To obtain these estimates a scale decomposition of the generation integral was attempted, two models for the release of latent heat were used, and an infrared cooling and solar absorption model formulated.

The following conclusions may be made:

1) The component of the generation integral associated with the hurricane's baroclinicity can be utilized to provide physically realistic estimates of the rate of production of energy available for conversion to kinetic energy within the hurricane scale.

2) The local generation of available potential energy by the latent heat component occurs mainly in the middle and upper troposphere within 200 km. of the hurricane center and not only depends on the sea surface temperature but also on the difference between the equivalent potential temperature of the boundary layer and the potential temperature distribution of the environment.

3) The local generation of available potential energy by the infrared component occurs primarily in the upper troposphere of the region beyond 500 km.

4) The generation for the hurricane scale by direct solar absorption and sensible heat addition is negligible. However, the dynamic role of the sensible heating in maintaining the deep cumulus convection is undoubtedly important.

5) The total generation estimate for Hilda using a sea surface temperature of 28°C. is 10.3×10^{12} watts with approximately 6 percent being contributed by direct solar absorption, 17 percent by infrared cooling, and 77 percent by the release of latent heat.

6) According to available potential energy considerations, the latent heat release necessary for the formation

of tropical storms should occur in a region of positive efficiency factors.

In future studies, detailed isentropic cross sections through several disturbances that develop into hurricanes should be made. Particular emphasis should be placed on the determination of the location where the intense precipitation and deep cumulonimbus convection develops in relation to the efficiency factor distribution.

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